

$C(A)$ & $N(A)$

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 6 & 1 & 5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$C(A) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} a + \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} b + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} c$$

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 1 & 3 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 1 & 3 \\ 0 & 0 & 2 & -1 \end{bmatrix}$$

$$N(A) =$$

$$\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{aligned} a &= 1 \\ b &= -1 \\ c &= -1 \\ d &= 1 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 10 & 20 & 30 \end{bmatrix} \rightarrow \begin{bmatrix} \textcircled{1} & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} \uparrow & \uparrow \\ b & c \end{matrix}$$

$$C(A) = \begin{bmatrix} 1 \\ 2 \\ 10 \end{bmatrix} a$$

$$a \quad -2 \quad -3$$

$$b \quad 1 \quad 0$$

$$c \quad 0 \quad 1$$

$$N(A) = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} x_2$$

$$ax + by = e$$

$$cx + dz = f$$

$$cax + cby = ce$$

$$acx + adz = af$$

$$(cb - ad)y = ce - af$$

$$y = \frac{ce - af}{cb - ad}$$